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THE EFFECT OF THERMAL DISTORTION ON CONSTRICTION RESISTANCE

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Abstract—A mathematical treatment is given of the effect of thermal distortion on the thermal contact resistance between two semi-infinite solids of different materials. Good agreement is achieved with experimental observations of thermal rectification due to Clausing. It is shown that problems of this type sometimes have no steady state solution and it is suggested that this is attributable to the discontinuous nature of the boundary condition for thermal contact. Expressions are given for the surface displacements due to steady state point and circular heat sources which could be of more general application.

NOMENCLATURE

- A, total area of actual contact;
- a, radius of contact area;
- a', radius of a cylindrical specimen;
- b, arbitrary limit of integration;
- c_0, $\dfrac{(1-\nu_1^2)}{E_1} + \dfrac{(1-\nu_2^2)}{E_2}$;
- c_1, c_2, $\dfrac{\alpha_1(1+\nu_1)}{K_1}, \dfrac{\alpha_2(1+\nu_2)}{K_2}$;
- E, Young's modulus;
- K, thermal conductivity;
- n, number of contact areas;
- p, pressure;
- Q, total heat flux;
- q, heat flux per unit area;
- R_0, $\dfrac{R_1R_2}{R_1+R_2}$;
- R_1, R_2, radii of curvature of the contacting surfaces when unheated;
- r, s, distance from the axis of symmetry;
- T, temperature;
- W, load;
- x, y, mutually orthogonal co-ordinates in the interfacial plane;
- z, co-ordinate perpendicular to x and y;
- α, coefficient of thermal expansion;
- λ, component of thermal strain at the surface perpendicular to the interfacial plane;
- ν, Poisson's ratio;
- ρ, thermal contact resistance;
- σ, direct stress.

Subscripts

- 0, interface or mean value;
- 1, 2, solids 1, 2;
- T, total value in a system with several contact areas.

1. INTRODUCTION

THE HEAT flow through the interface between two apparently conforming solids is less than that which would be obtained through a single solid of the same overall shape and size and with the same temperatures at the boundaries. This is due to the inevitable roughness of the surfaces, which prevents the solids from making actual contact except at a few small areas within the apparent contact area. If the solids are good thermal conductors, most of the heat will flow through these actual contact areas in preference to passing between the non-contacting parts of the interface by radiation or conduction through the intervening gas (if any). This distortion of the heat flow causes an increase in thermal resistance which is known as the *thermal contact resistance* or *constriction resistance*. This phenomenon has been the subject of intensive study in recent years because of its relevance to the conduction of heat from nuclear fuel elements.

It has sometimes been noted that the contact resistance between dissimilar metals depends on the direction of heat flow. This process has become known as *thermal rectification* (by analogy with electrical rectification at semi-conductor interfaces) and various theories have been proposed to account for it [1–6]. One of the more convincing explanations was suggested by Clausing [6] and depends on the influence of thermal strain on the contact conditions. Suppose heat is conducted across an interface from solid 1 to solid 2. The input of heat to solid 2 is not uniformly distributed, but occurs primarily at areas of actual contact. Thus, the material near to the actual contact areas will be hotter than at other parts of solid 2 and differential thermal expansion will occur, forcing the bulk of the solids apart. In the absence of any comparable effect in solid 1, this would tend to reduce the contact area and hence increase the constriction resistance. However, the flow of heat in solid 1 is similarly non-uniform except that the *outflow* of heat is concentrated at the actual contact areas which are therefore *cooler* than the rest of the solid. Thus, for solid 1, the process is reversed; the heat flow causes the contact regions to contract, allowing the solids to move together. The distribution of heat output from solid 1 must be equal to that of heat input to solid 2, but, if the solids are made of different materials, the thermal strains will generally differ and a change in constriction resistance will occur. If the contraction of solid 1 is greater than the expansion of solid 2, the solids will move closer together, further areas of the surface will be brought into contact and the constriction resistance will fall. If the same heat flow were passed through the interface in the opposite direction, the thermal strains would all change sign and the solids would be forced apart causing an increase in constriction resistance. If the solids are made of the same materials, the thermal strains will be equal and opposite and no change in contact conditions or constriction resistance will occur.

It is generally found that the actual contacts between solids are not uniformly distributed over the apparent contact area, but are grouped into regions (known as *contour areas*) because of the existence of long wavelength roughness ("waviness") in the machined surface [7]. The contour areas constitute an additional source of constriction resistance which Clausing [6, 8] claimed could be dominant in many thermal contact systems. He therefore carried out a series of experiments with contacting dissimilar solids of known waviness and found a considerable dependence of constriction resistance on the direction of heat flow. At low values of heat flow, when the rectification effect was small, the observed contact resistance was close to the value predicted theoretically from the known waviness of the profile on the assumption that the resistance due to constriction of the heat flow through the contour area was dominant (i.e. that there was perfect thermal contact between the solids throughout the contour area). However, Clausing did not extend his analysis to those cases where thermal strains were significant, but discussed these results in purely qualitative terms.

In this paper, the effect of thermal strain on contact resistance is analysed for a number of particular systems with rotational symmetry. The results of this analysis are compared with Clausing's experimental results in section 7 and a number of unusual features in the analysis are discussed in greater detail in sections 9, 10.

2. METHOD OF SOLUTION

In this analysis, we shall assume that all deformation is elastic and that the solids are in perfect thermal contact throughout the elastic contact area. The agreement between theory and experiment obtained by Clausing on the basis of these assumptions suggests that they are acceptable for his experimental system, but the implications of different contact conditions will be discussed in section 8.

Suppose that two semi-infinite solids with spherical surfaces are loaded against each other. In the absence of thermal strains, we can find the radius of the contact area, the distribution of pressure and the elastic displacements as functions of the applied load and the properties of the solids. However, if the extremities of the solids are maintained at different temperatures, heat will flow through the contact area causing thermal distortion which will itself affect the contact area.

In order to solve this problem we shall assume that the final contact area is a circle of radius a. We can then find the steady state temperature at all points in the solids on the assumption that no heat flows across the interface except through the contact area. This temperature field will cause certain thermal strains which, in the absence of contact forces would distort the surface profiles Thus, by finding these thermal strains we can finally find the load which must be applied between the distorted solids to establish a contact area of the assumed size. This method gives a particular solution, the uniqueness of which will be discussed in section 6.

3. THE POINT SOURCE SOLUTION

It is convenient to find first the thermal strains produced by a point continuous heat source of strength q on the surface of a semi-infinite solid. This solution can then be extended by suitable integration.

The temperature T at a distance r from the source is

$$T = \frac{q}{2\pi K r} \tag{1}$$

where K is the thermal conductivity of the solid [9]. The thermal strains due to this temperature distribution can be obtained by considering the equivalent infinite solid problem and superposing surface forces equal and opposite to the stresses transmitted across the surface plane. This method is described in greater detail in [10]. Thus, if equation (1) represented the temperature distribution in an infinite solid, the direct stress tangential to the sphere of symmetry (and hence normal to any plane including the heat source) would be

$$\sigma = \frac{\alpha E}{(1-\nu)}\left\{\frac{1}{r^3}\int_0^r T r^2 \, dr - T\right\} \tag{2}$$

where α is the coefficient of thermal expansion E is Young's modulus and ν is Poisson's ratio (see TG: Art136).*

i.e.

$$\sigma = -\frac{q\alpha E}{4\pi(1-\nu)Kr}. \tag{3}$$

The surface displacements of the semi-infinite solid can be found by superposing a surface pressure equal and opposite to the stress σ, since the symmetry of the infinite body problem precludes the possibility of shear stresses being transmitted across any plane including the heat source.

The displacement (λ) on the surface of a semi-infinite solid due to a point load W at a distance r is

$$\lambda = \frac{W(1-\nu^2)}{\pi E r} \tag{4}$$

(TG: Art. 123). Thus, the normal surface displacement at a radius s due to the pressure distribution equal and opposite to that given by equation (3) is

$$\lambda = -\int_0^{\infty}\int_0^{2\pi} \frac{\sigma(1-\nu^2) r \, dr \, d\theta}{\pi E \sqrt{(r^2 - 2rs\cos\theta + s^2)}}$$

$$= \frac{q\alpha(1+\nu)}{4\pi^2 K}\int_0^{\infty}\int_0^{2\pi} \frac{dr \, d\theta}{\sqrt{(r^2 - 2rs\cos\theta + s^2)}}. \tag{5}$$

This integral does not approach a limit as r tends to infinity. However, this is a consequence of the assumption that the solids are semi-infinite. An essentially similar difficulty is encountered in

* Specific sections of references [11] and [12] are cited in this form.

two-dimensional potential flow and contact problems and it can be avoided by replacing the infinite upper limit to the integral by the finite quantity b such that $b \gg s$. In this case, the integral can be evaluated to give

$$\lambda = \frac{q\alpha(1 + \nu)}{2\pi K}\{\log(4b) - \log(s)\} \qquad (6)$$

(see Appendix I). It should be noted that the term including the arbitrary limit b merely constitutes a bulk displacement of the surface and does not affect the profile. The value of b will not therefore affect the contact problem.

4. HEAT FLOW THROUGH A CIRCULAR CONTACT AREA

The heat input to solid 2 must be equal to the heat output from solid 1 and hence the temperature fields in the two solids must be similar (i.e. isothermal surfaces will be symmetrical about the interfacial plane, but the actual temperature difference between any two pairs of corresponding isothermal surfaces will depend on the relationship between the conductivities of the materials). In particular, the temperature must be continuous through the contact area and hence the latter must be an isothermal surface whose temperature is

$$T_0 = \frac{K_1T_1 + K_2T_2}{(K_1 + K_2)} \qquad (7)$$

where K_1, K_2, T_1, T_2, are the conductivities and "temperatures at infinity" respectively in solids 1 and 2. This result applies to any shape of contact area or distribution of contact areas between geometrically symmetrical solids.*

* This result is completely general and independent of the assumptions of elastic deformation and perfect thermal contact provided that the term "contact area" is taken to mean any area which is in actual metallic contact. If some of the heat flow between the solids takes place at areas which are not in actual contact (i.e. through surface films or an intervening gas), these areas will not be at the temperature T_0, although this temperature must occur at some point in the intervening material.

Thus, the distribution of heat flow through the contact area must be such as to maintain the contact area at the uniform temperature T_0. It can be shown that a temperature difference T between a circular area on the surface and the extremities of a semi-infinite solid causes a heat input (q) per unit area at a radius r given by

$$q = \frac{2KT}{\pi\sqrt{(a^2 - r^2)}} \qquad (8)$$

where $0 < r < a$. The derivation of the mathematically similar result for contact pressures is is given in TG: Art. 124.

Thus, the heat flow per unit area through the contact area is obtained by putting T equal to $T_0 - T_2$, i.e. from equations (7) and (8),

$$q = \frac{K_0(T_1 - T_2)}{\pi\sqrt{(a^2 - r^2)}} \qquad (9)$$

where

$$K_0 = \frac{2K_1K_2}{(K_1 + T_2)}. \qquad (10)$$

The surface displacements (λ_2) in solid 2, at a radius s, due to this heat input can be found by substituting for q in equation (6) and integrating over the contact area. Thus,

$$\lambda_2 = \frac{c_2(T_1 - T_2)K_0}{2\pi^2} \int_0^a\int_0^{2\pi} \frac{\{\log(4b) - \log[\sqrt{(s^2 - 2rs\cos\theta + r^2)}]\}\,r\,dr\,d\theta}{\sqrt{(a^2 - r^2)}} \qquad (11)$$

where

$$C_2 = \frac{\alpha_2(1 + \nu_2)}{K_2}. \qquad (12)$$

This integral can be evaluated (see Appendix II) to give

$$\lambda_2 = \frac{c_2(T_1 - T_2)K_0 a}{\pi}\left[\left[\log\left\{\frac{4b}{s}\left[\frac{a}{s} - \sqrt{\left(\frac{a^2}{s^2} - 1\right)}\right]\right\} + \sqrt{\left(1 - \frac{s^2}{a^2}\right)}\right]\right] \tag{13}$$

for $0 < s \leqslant a$ and

$$\lambda_2 = \frac{C_2(T_1 - T_2)K_0 a}{\pi}\log\left(\frac{4b}{s}\right) \tag{14}$$

for $s > a$. The corresponding displacements in solid 1 can be found by transposing the subscripts 1 and 2 in equations (13) and (14).

The total heat flow rate (Q) through the contact area can be obtained by integrating equation (9) over the contact area and is

$$Q = 2K_0(T_1 - T_2)a. \tag{15}$$

It is worth noting that if we approximate the heat flow through the contact area to the average value ($Q/\pi a^2$), the corresponding value of λ_2 for $0 < s \leqslant a$ (derived in Appendix II) is

$$\lambda_2 = \frac{C_2(T_1 - T_2)K_0 a}{\pi}\left\{\log\left(\frac{4b}{a}\right) + \frac{1}{2}\left(1 - \frac{s^2}{a^2}\right)\right\}. \tag{16}$$

This approximate solution is compared with the exact solution in Fig. 1. In the range $s > a$, the two solutions are identical, but, within the heated area, a uniform heat input causes greater distortion than the distribution given by equation (9). However, in complex systems, where an exact solution is impossible, a satisfactory approximation might be obtained by distributing the total heat input uniformly over the contact area.

5. PRESSURE DISTRIBUTION AT THE CONTACT AREA

To complete the solution we have to find the distribution of pressure necessary to cause the thermally distorted solids to conform throughout the contact area. For this purpose, any bulk expansion of the solids is irrelevant and the profile given by equation (13) can be conveniently referred to the point $s = 0$. The equation of the distorted surface related to this point as origin is

$$z_2 = \lambda_2(0) - \lambda_2(s) \tag{17}$$

where the co-ordinate z_2 is measured perpendicularly away from the interfacial plane.

If the undistorted surface of solid 2 is a sphere

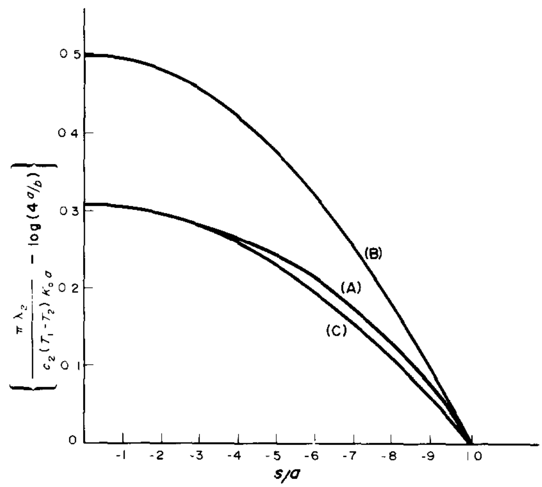

FIG. 1. Graphical representation of the surface distortion produced by (A) a constant temperature [equation (14)] and (B) a uniform heat input at a circular area radius a [equation (16)]. Curve (C) shows the parabolic approximation to (A) which is used in equation (19).

of radius R_2 within the contact area, the equation of the distorted surface will be approximately modified to

$$z_2 = \frac{s^2}{2R_2} + \lambda_2(0) - \lambda_2(s). \tag{18}$$

To obtain an analytical solution to the contact problem, the surface defined by equation (18) can be approximated to the paraboloid passing through $z_2(0)$ and $z_2(a)$, the equation of which is

$$z_2 = \left\{\frac{1}{2R_2} + \frac{c_2(T_1 - T_2)K_0(1 - \log 2)}{\pi a}\right\} s^2 \quad (19)$$

and the corresponding equation for solid 1 is

$$z_1 = \left\{\frac{1}{2R_1} + \frac{c_1(T_2 - T_2)K_0(1 - \log 2)}{\pi a} s^2\right\}. \quad (20)$$

The extent of the approximation involved here can be assessed by comparing curves (A) and (C) in Fig. 1.

The pressure distribution (p) needed to make the surfaces defined by equations (19) and (20) conform throughout the contact circle is

$$p = \frac{4}{\pi c_0} \left\{\frac{1}{2R_0} + \frac{(c_2 - c_1)(T_1 - T_2)K_0(1 - \log 2)}{\pi a}\right\} \sqrt{(a^2 - s^2)}, \quad (21)$$

where

$$c_0 = \frac{(1 - \nu_1^2)}{E_1} + \frac{(1 - \nu_2^2)}{E_2},$$

$$R_0 = \frac{R_1 R_2}{(R_1 + R_2)}$$

(see TG; Art. 125). The total load is obtained by integrating equation (21) over the contact area and is

$$W = \frac{4a^3}{3R_0 c_0} + \frac{8(c_2 - c_1)(T_1 - T_2)K_0(1 - \log 2)a^2}{3\pi c_0}. \quad (22)$$

The uniform heat flow equation (16) already has the necessary parabolic form and leads to the result

$$W = \frac{4a^3}{3R_0 c_0} + \frac{4(c_2 - c_1)(T_1 - T_2)K_0 a^2}{3\pi c_0}. \quad (23)$$

Both these equations (22) and (23) reduce to the usual Hertzian relation for the contact of two spheres if $T_1 = T_2$ (no heat flow) or $c_1 = c_2$ (similar materials).

6. UNIQUENESS OF THE SOLUTION

Equation (22) gives the load (W) required to cause the solids 1, 2 to conform over a circular area of radius a, when their extremities are maintained at temperatures T_1, T_2 respectively. However, we can equally regard a as the dependant variable in the equation provided that the uniqueness of the solution can be established.

The surface displacements in a semi-infinite solid due to heating and normal loading satisfy a modified form of Laplace's equation, but the uniqueness theorem is not valid for contact problems because of the influence of displacement on the boundary conditions for heat flow. In fact, if $c_2 > c_1$, there will generally be several steady state solutions. For example, consider two solids which make contact at a number of well separated contact areas when the heat flow is zero and suppose that, before the solids are placed in contact, heat is supplied to solid 2 through one of these areas only, causing local expansion. This distortion will restrict the initial contact of the solids to the previously heated area and the heat flow through this area from solid 1 will tend to perpetuate the condition. Thus, if $T_1 - T_2$ is sufficiently large, there will be a separate state solution for each original contact area. As $T_1 - T_2$ is reduced, the number of solutions will also fall until, when $T_1 = T_2$, there is only one solution, since with no heat flow, the problem reduces to one of solid contact and the potential theory uniqueness theorem applies. There remains one stable solution for further reduction of $T_1 - T_2$ into the negative range [i.e. $(c_2 - c_1)(T_1 - T_2) < 0$] until a certain critical temperature difference is reached beyond which there is no steady state solution. This result is discussed in section 10.

For the axisymmetric problem considered in section 5, we can conveniently restrict our attention to axisymmetric solutions on the grounds that the initial boundary conditions are axisymmetric. It can be shown from considerations of displacement curvature that the only axisymmetric steady state solution for heat flow in either direction involves a single circular contact area and hence equation (22) defines a unique solution to the problem under discussion.

7. THE EFFECT ON THERMAL CONTACT RESISTANCE

The thermal contact resistance (ρ) is defined as the temperature difference ($T_1 - T_2$) per unit heat flow (Q). i.e.

$$\begin{aligned}\rho &= (T_1 - T_2)/Q \\ &= \tfrac{1}{2}K_0 a \qquad (24)\end{aligned}$$

from equation (15). Thus, the relationship between the applied load (W), the thermal contact resistance (ρ) and the heat flux (Q) can be found from equations (15), (22) and (24) and is

$$\frac{2(1 - \log 2)(c_2 - c_1)Q}{\pi} + \frac{1}{2R_0 K_0^2 \rho^2} = 3c_0 W K_0 \rho. \qquad (25)$$

The relation between ρ and Q for particular fixed values of the other parameters is shown in Fig. 2. The figure is presented in dimensional terms despite the consequent loss of generality, since the influence of load and initial curvature are thereby separated. For large positive values of heat flux, the thermal contact resistance approaches asymptotically to the linear relation

$$\rho K_0 = \frac{2(1 - \log 2)(c_2 - c_1)Q}{3\pi c_0 W} \qquad (26)$$

which is shown dotted in Fig. 2. The slope of this line is inversely proportional to $c_0 W$, but independent of the initial radius of curvature of the surfaces (R_0). For large negative values of Q (i.e. large heat fluxes from solid 2 to solid 1), the contact resistance tends to zero. Figure 2 only represents equation (25) for the case where $c_2 > c_1$, but the direction of positive heat flux (Q) can always be defined to satisfy this condition. Alternatively, it is easily verified from equation (25) that the interchange of materials of solids 1 and 2 merely changes the sign of ($c_2 - c_1$) and is therefore equivalent to a reversal of heat flow direction as one would expect. The other terms in equation (25) only contain mean values of

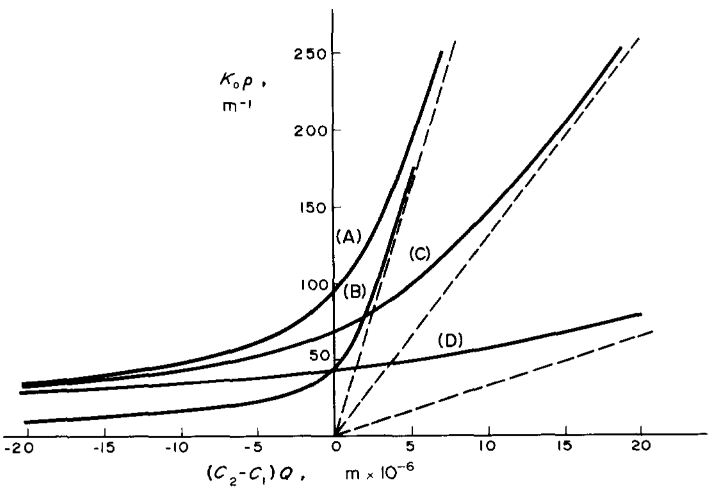

FIG. 2. The effect of heat flux (Q) on contact resistance (ρ) from equation (26) for $c_0 W = 2 \times 10^{-9}$ m (A, B), 5×10^{-9} m (C), 20×10^{-9} m (D) and $R_0 = 100$ m (A, C, D), 1000 m (B).

material properties and are therefore unaffected by an interchange of materials.

The experimental dependence of contact resistance on heat flow observed by Clausing [6] using an aluminium–stainless steel interface is shown in Fig. 3. As originally presented, points corresponding to negative values of Q were placed in the positive quadrant, the direction of heat flow being noted. Two curves of resistance against heat flow were therefore obtained for each load, these being extrapolated to a single point on the axis $Q = 0$. In Fig. 3, Q is taken as

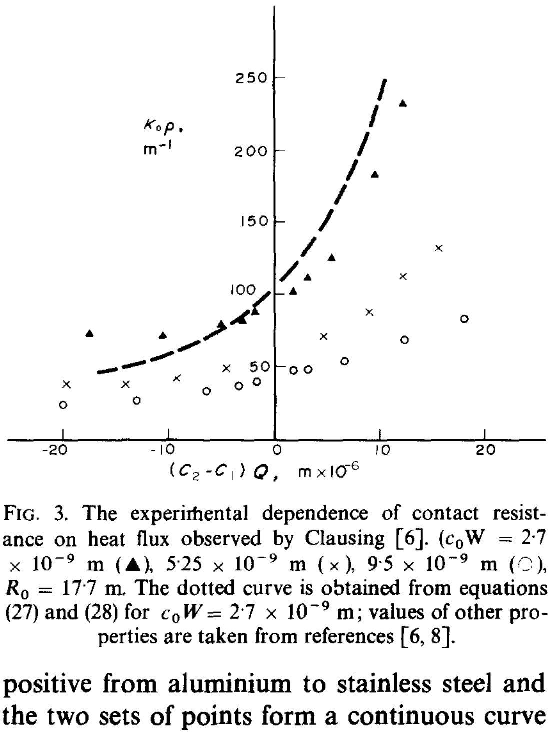

FIG. 3. The experimental dependence of contact resistance on heat flux observed by Clausing [6]. ($c_0W = 2{\cdot}7 \times 10^{-9}$ m (▲), $5{\cdot}25 \times 10^{-9}$ m (×), $9{\cdot}5 \times 10^{-9}$ m (○), $R_0 = 17{\cdot}7$ m. The dotted curve is obtained from equations (27) and (28) for $c_0W = 2{\cdot}7 \times 10^{-9}$ m; values of other properties are taken from references [6, 8].

positive from aluminium to stainless steel and the two sets of points form a continuous curve crossing the axis $Q = 0$ for each load.

Clausing's experimental system consisted of two cylindrical specimens with spherical contact surfaces. The contour area was significant in comparison with the cylinder cross section so that a direct comparison between Figs. 2 and 3 is not strictly valid. However, one would expect the semi-infinite solid to exhibit the same qualitative behaviour as the finite cylinder and this prediction is confirmed. The two sets of curves (Figs. 2 and 3) are similar in shape and the slopes of the asymptotic lines at high positive heat flux are found to be approximately inversely proportional to load as predicted.

The heat flow through the contact area between two cylinders spreads out within the solids until it is uniformly distributed over the cross section. Thus, the contact resistance and thermal distortion are due to the *difference* between the actual heat flow conditions and those of uniform heat flow through the interface. We can therefore obtain an approximate solution to this problem by applying the same *differential* heat flow conditions to the semi-infinite solids. Thus, the thermal distortion produced at the surface of a cylinder radius a' by a heat input Q over a circle radius a ($a < a'$) is assumed to be the same as that produced on the surface of a semi-infinite solid in which there is a heat input Q at the contact circle, radius a, and a heat *output* Q uniformly distributed over a concentric circle radius a'. The modified distortion equation for this problem can be found from equations (13) and (16) and leads to the relation

$$W = \frac{4a^3}{3R_0c_0} + \frac{2(c_2 - c_1)Q}{3\pi c_0} \left\{2(1 - \log 2) - \left(\frac{a}{a'}\right)^2\right\}. \quad (27)$$

An approximate value for the contact resistance (ρ) for the cylinder radius a' can be found by a similar argument and is

$$\rho = \frac{1}{2K_0a} - \frac{2}{\pi K_0a'}. \quad (28)$$

A number of more precise numerical solutions have been obtained for this latter problem, the results of which confirm the validity of equation (28) in the range $a/a' < \frac{1}{2}$.

By eliminating the unknown contact area radius (a) from equations (27) and (28) a relationship between ρ and Q is obtained, which is shown dotted in Fig. 3 for the conditions of Clausing's experiments at a load of 157N;

values of the material properties and the surface curvature are taken from references [6, 8]. Reasonable agreement is achieved between theory and experiment.

8. THE MICROSCOPIC CONTACT CONDITIONS

The effects of thermal strain have so far only been discussed with reference to systems in which the solids are in perfect thermal contact throughout the elastic contact area. In practice, there will be a number of microscopic actual contacts distributed over this area and separated by non-contacting regions.

The resistance to heat flow through such a group of actual contacts can be regarded as the sum of two parts; the microscopic resistance, which is the resistance which would be obtained if all the existing actual contacts were widely separated, and the macroscopic contact resistance, which is the additional resistance due to the grouping of the actual contacts within a finite contour area. For a wide range of distributions of actual contacts, the macroscopic contact resistance, as defined above, is approximately equal to the resistance to heat flow between two solids which are in perfect thermal contact throughout the contour area. The validity of this approximation is considered in greater detail by Greenwood [13].

Thus, the analysis given in sections 2–7 can be applied to a system in which the contact is discontinuous, providing that the microscopic constriction resistance is small in comparison with the macroscopic or contour area resistance.

If the microscopic resistance is significant, the total resistance will contain an additional term which must be found from a consideration of the actual contact conditions. Early theories of solid contact were based on the assumption that plastic deformation occurs at the areas of actual contact and that the total area of actual contact (A) is therefore linearly related to the total load. i.e.

$$W = Ap \tag{29}$$

where p is the normal pressure at the actual contact areas and is tentatively identified with the indentation hardness of the softer material.

It is clear that, if this theory were true, thermal strains would have no effect on the total area of actual contact. Thus, if the average size of the actual contacts remained constant, the only possible effect on contact resistance would be caused by a redistribution of actual contact areas over the apparent contact area, with a consequent change in the contour area resistance. At high negative heat fluxes, the load and consequently the actual contact areas would be almost uniformly distributed over the apparent contact area and the total contact resistance would tend to a finite limit. The value of this limit is the microscopic contact resistance and it would fall with increasing load since it depends on the total area of actual contact (A) given by equation (29). Clausing's experimental results show just this tendency (see Fig. 3) and the limits of contact resistance at high negative heat fluxes are approximately inversely proportional to load, thus lending some support to a "plastic" theory of contact.

On the other hand, recent theories of contact have shown that much of the evidence in favour of plastic contact conditions (such as the proportionality between tangential and normal forces in sliding) can be explained as a consequence of the geometrical properties of the solid surface [14, 15]. When two newly prepared solids are placed in contact, it is possible that much of the original deformation will be plastic, but work hardening will occur near the actual contact areas and one would expect any subsequent contact deformation to be primarily elastic. If this is so, the microscopic constriction resistance will be affected by thermal strain. However, there are two reasons for supposing that this effect will be small compared with the effect on the contour area. Firstly, the radius of curvature [R_0 in equation (25)] is generally much larger for the waviness of the surface than for the microscopic roughness. Thus, more thermal distortion is necessary to produce the same proportionate change in contact resistance. Secondly, fragmentation of the contact area tends to reduce the effect of thermal strain. Suppose there are n widely separated contact areas of equal thermal resistance (ρ), each carrying a load W and transmitting a heat flux Q, these quantities being related by equation (25). The total heat flux ($Q_T = nQ$), the total load ($W_T = nW$) and total thermal resistance ($\rho_T = \rho/n$) are therefore given by

$$\frac{2(1-\log 2)}{\pi}(c_2 - c_1)\frac{Q_T}{n} + \frac{1}{2R_0K_0^2\rho_T^2n^2} = 3c_0W_TK_0\rho_T. \quad (30)$$

From this equation, it follows that, for given values of Q_T, W_T and ρ_T, $(\partial\rho_T/\partial Q_T)$ falls with increasing n. It should, however, be noted that the problem has been oversimplified by assuming that the n contacts are of equal size and widely separated. In practice, these conditions will not be met; any difference in size between contact areas will be exaggerated by a positive heat flux and interaction between the distortion fields of adjacent contacts also affects the pressure distribution. However, these effects occur on the scale of the contour area and are covered, albeit approximately, by an analysis of the latter.

Lewis and Perkins [16] have recently described some experiments in which the thermal rectification effect was found to fall when the waviness of the surfaces was reduced. A surprising feature of these experiments is that, when the waviness and microscopic roughness were comparable, a reversed rectification effect was observed; i.e. the contact resistance was found to be higher for heat flow from stainless steel to aluminium. A reversed rectification effect was also observed by Barzelay *et al.* [1] and was attributed by Clausing [6] to thermal distortion induced by the existence of radial temperature gradients in the cylindrical specimens. However, Lewis and Perkins [16] and more recently Thomas and Probert [17] claim to have eliminated heat losses from the sides of the specimen and they still found a reversed rectification. Lewis and Perkins proposed that this phenomenon could be attributed to the effect of thermal strains on the miscroscopic scale, particularly if surface discontinuities exist in the contact region. However, a number of difficulties arise with this theory, some of which must also be considered in connection with the analysis developed in section 3–5.

9. THE EFFECT OF SURFACE DISCONTINUITIES

Consider a region of actual contact between two dissimilar metals and suppose that a discontinuity exists in one of the surfaces as shown in Fig. 4a. The heat flow through this interface may be considered as the sum of a uniform positive heat flux over the entire interface and a negative heat flux through the non-contacting area at the discontinuity, the latter component being chosen so as to make the total heat flux through this area zero. Providing the actual contact region is large compared with the dimensions of the discontinuity, the uniform heat flux through the interface will produce bulk expansion only and no distortion of the surface. Thus the distortion of the surface will be the same as that due to a negative heat flux through the non-contacting region. If the net heat flow is from solid 1 to solid 2 (where $c_1 > c_2$), the distortion produced will be as shown in Fig. 4b.

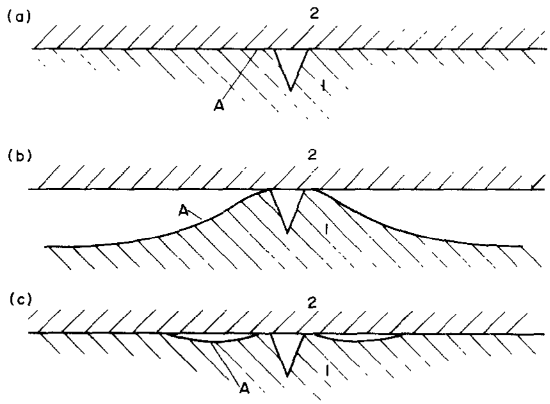

FIG. 4. The effect of a discontinuity in contact conditions, (a), Geometry for zero heat flow, (b, c), Transient response and Equilibrium state respectively proposed by Lewis and Perkins [15] for heat flow in the direction ($1 \to 2$) where ($c_1 > c_2$).

However, it is clear that this cannot be an equilibrium condition since the heat flow through the interface will be affected by the changed contact conditions. Lewis and Perkins therefore propose that the system will eventually reach an equilibrium state such as that shown in Fig. 4c, where part of the original contact area is now out of contact. This would cause an increase in thermal resistance for the opposite direction of heat flow to that considered in section 5.*

However, Fig. 4c cannot represent an equilibrium state. Consider the point A which is in contact before the heat flow occurs, but which is supposed to be removed from contact by thermal strain. The relative normal displacement $[\lambda = -(\lambda_1 + \lambda_2)]$ of A due to thermal strain must be greater than that at any point which remains in contact. Thus, if Fig. 4c represents a possible equilibrium state, the point of maximum relative displacement (not necessarily A) must lie in a region which is not in contact and hence at which there is no heat flow. At the point of maximum displacement,

$$\left(\frac{\partial_2\lambda}{\partial x^2}\right) + \left(\frac{\partial_2\lambda}{\partial y^2}\right) < 0 \qquad (31)$$

where x and y are any two mutually orthogonal co-ordinates in the interfacial plane. But it is easily verified that for the point source solution [equation (6)], the left-hand side of the inequality (31) is equal to zero everywhere except at the source ($s = 0$). Hence any integration of equation (6) will give a profile $[\lambda(x,y)]$ such that

$$\left(\frac{\partial_2\lambda}{\partial x^2}\right) + \left(\frac{\partial_2\lambda}{\partial y^2}\right) = 0 \qquad (32)$$

throughout any unheated area. The maximum value of λ cannot therefore occur in such an area, nor can it occur at the remote boundaries of the solid, since for heat flow in the direction 1 to 2 where $c_1 > c_2$, λ decreases with distance from the contact area provided that this distance is sufficiently large. This is actually a particular two-dimensional case of a general result in potential theory.

If a load is transmitted between the solids, the above argument still applies, provided that point A was in contact in the unloaded state, since, by a similar argument from equation (4), the inequality (31) cannot be satisfied in a non-loaded area.

Thus, with heat flow in the direction 1 to 2, the solids cannot reach an equilibrium state with a reduction in contact area, but, as shown by Lewis and Perkins, neither can they remain in equilibrium without a change in contact area. This implies that either the behaviour of the system will be time dependent or that the controlling equations are inherently indeterminate under certain circumstances. In order to investigate this matter further it is helpful to consider some particular examples.

10. NON-EQUILIBRIUM CONDITIONS

If two perfectly flat solids are placed in contact, the relationship between heat flux and contact resistance [equation (26)] reduces to

$$2(1 - \log 2)(c_2 - c_1)Q = 3\pi c_0 W K_0 \rho. \qquad (33)$$

This equation gives a realistic result if $(c_2 - c_1)Q$ is positive, but it implies that the contact resistance is negative for reversed heat flow. However, this is a spurious result which is a result of the existence of an infinite contact area. If instead of putting $1/R_0 = 0$, we allow it to approach zero asymptotically, the curve in Fig. 2 approaches the line given by equation (32) for large positive values of $(c_2 - c_1)Q$ and the axis $K_0\rho = 0$ for large negative values.

If the apparent contact area is restrained to a finite value by the surface profile of one of the solids, there are some loading conditions for which no equilibrium solution is possible. Thus,

* For a more detailed development of this theory see Lewis and Perkins [16]. They claim that this effect will only be produced if the discontinuity is in the solid of lower thermal conductivity, but this is not necessary. The temperature fields, thermal and elastic strains and contact pressures are only affected by the relative profiles of the surfaces to a first approximation. Thus, the argument is equally applicable to the geometry shown in Fig. 2(b) of reference [16].

suppose the solids are initially flat and the contact area is geometrically restricted to a circle of radius a by machining a step in one surface. It was proved in section 9 that if heat flows from solid 1 to solid 2, where $c_1 > c_2$, no steady state solution is possible in which any part of the original contact area was removed from contact by thermal strain. Thus, if a solution exists, heat flow must take place through the contact circle radius a and the temperature field in the solids is similar to that considered in section 4. Thus, the thermal strains will also be similar and the pressure distribution needed to maintain the contact over the entire circle is obtained from equation (21) by putting $1/R_0 = 0$. i.e.

$$P = \frac{4(1 - \log 2)(c_2 - c_1)(T_1 - T_2) K_0 \sqrt{(a^2 - s^2)}}{\pi^2 c_0 a}. \quad (34)$$

However, $(T_1 - T_2)(c_2 - c_1)$ is now negative so that tensions must be applied between the surfaces to maintain them in conformity. This is not an acceptable solution, but we can find one by superimposing a pressure distribution chosen so as to cause an equal displacement at all parts of the contact circle and of such value as to make the total pressure just positive at all points.

The pressure distribution

$$p' = \frac{C}{\sqrt{(a^2 - s^2)}} \quad (35)$$

over the contact area produces an equal displacement at all points $0 \leqslant s \leqslant a$ [see TG: Art. 124 and cf. equation (8)], where C is a constant. The minimum positive pressure distribution to achieve conformity will occur when the constant C is chosen to make the minimum value of $(p' + p)$ equal to zero. This minimum pressure occurs at $s = 0$, hence

$$\frac{C_{\min}}{a} = \frac{4(1 - \log 2)(c_1 - c_2)(T_1 - T_2) K_0}{\pi^2 c_0}. \quad (36)$$

For this value,

$$p + p' = \frac{4(1 - \log 2)(c_1 - c_2)(T_1 - T_2) K_0}{\pi^2 c_0} \left\{ \frac{a}{\sqrt{(a^2 - s^2)}} - \frac{\sqrt{(a^2 - s^2)}}{a} \right\}. \quad (37)$$

The integral of equation (37) over the contact area gives the minimum load (W) which will cause the solids to conform. i.e.

$$W = \frac{16(1 - \log 2)(c_1 - c_2)(T_1 - T_2) K_0 a^2}{3\pi c_0}. \quad (38)$$

If the load is greater than this value, the constant C in the pressure component p' will be increased causing a greater bulk deformation, but maintaining conformity between the solids for $0 \leqslant s \leqslant a$. However, for loads less than W, there is no distribution of positive pressure which will achieve conformity and hence no solution is possible.

It is clearly theoretically possible to write down a set of integro-differential equations for this system relating the past history of heat input to the instantaneous temperature field and the latter to the thermal strains and the consequent contact pressure distribution. These equations would be adequate to determine the dynamic behaviour of the system provided that suitable boundary conditions were specified (e.g. that the two solids, initially at uniform, but different, temperatures, are pressed together with a constant force after time $t = 0$). The existence of this determinate set of equations, one of which (the relation between temperature and heat flow) contains time dependent terms, suggests that a time dependent solution could be obtained for values of the contact force less than that given by equation (38). Presumably such a solution would show that any given point in the contact circle would experience periods in contact and periods out of contact. Periodic variations in contact resistance were observed by Clausing

(18) at low normal loads and it is tempting to attribute them to this mechanism. However, the observed period was long (1–2 h) in comparison with the time needed to reach a steady temperature in the specimens (15–20 min). Furthermore, there are systems for which even a time dependent solution is impossible. For example, consider a "solid" made up of two thermally insulated rods, of slightly differing lengths, clamped together at one end. If this solid is constrained to move in the direction of the rod axes and is lightly loaded against a cooler surface, contact will initially occur on the longer rod only, but the latter will contract until the second rod makes contact. However, the heat flow from rod 2 will be initially higher than that from rod 1, causing the contact to break an infinitesimal time after it has been made. This process will be repeated indefinitely. Thus, the solution for the transient heat flow from rod 2 to the other solid is a periodic function of infinite frequency with a finite mean value.

This is not an acceptable solution, but it demonstrates that the difficulty arises from the fact that an infinitesimal displacement makes the difference between contact and non-contact conditions. In a practical system, this will not be true; even if there is no intervening gas and the surfaces are uncontaminated, a finite local displacement must be necessary to effect the contact. This "contact displacement" may be completely negligible in comparison with the surface displacements due to contact pressures and thermal expansion, but it introduces the possibility of conditions intermediate between the extremes of infinite and zero local contact resistance. With this altered boundary condition, a stable solution can be found in which rod 2 is in imperfect contact with the other solid. Thus, the heat flow through rod 2 will be less than that through rod 1 and the thermal strain will be less. The actual value of heat flow will be determined by the condition that the difference in thermal contraction must be equal to the original difference in length of the rods.

In general, areas which would be removed from contact by thermal distortion will tend to reach an equilibrium state in which a finite local thermal resistance reduces the local heat input to the value necessary to maintain this imperfect contact condition. In a practical contact situation, it follows that, for loads less than the critical value, there will be some areas of imperfect contact and hence the contact resistance will be increased. Thus, with this modification of the boundary conditions of the problem, a reversed rectification effect could be produced. However, it is doubtful whether an experimental system would ever have the required geometry on the microscopic scale, since the effect depends on the existence of extensive actual conformity of the solids in the unloaded state. This is unlikely to occur. It is more probable on the "contour area" scale, in which case the areas of imperfect contact would be those in which the density of actual contacts was low. However, Lewis and Perkins' experimental results show that reversed rectification becomes more pronounced at high nominal pressures, whereas the theory developed above predicts that the effects should only be observed *below* a certain critical load, such as that given by equation (38) for the circular contact system. This suggests that the experimental evidence cannot be entirely explained as a consequence of thermal distortion. Furthermore, Williams [19] and Thomas and Probert [17] have observed directional effects in the thermal resistance between solids differing in surface geometry, but of similar materials. Unless radial temperature gradients existed, these results could not have been caused by thermal distortion. They are attributed by Thomas and Probert to the potential barrier produced by the interfacial oxide layers, this explanation of thermal rectification being originally suggested by Moon and Keeler [5]. It therefore appears that at least two independent mechanisms are involved in existing observations of thermal rectification.

11. CONCLUSIONS

This paper has been primarily concerned with developing an analysis of the interaction between thermal strain and interfacial thermal resistance. The general method of approach, outlined in section 2, could be applied to a wide range of problems and particular solutions are here obtained for the case of two large solids with axisymmetric curved surfaces in good thermal contact at a circular area. The theory can be applied to the contact of rough solids, providing that the microscopic constriction resistance is small in comparison with the contour area resistance. It is found that the contact resistance depends on the heat flow between the solids, the relationship being given by equation (25). This relationship is plotted in Fig. 2 and shows a striking qualitative agreement with experimental results due to Clausing [6] for a similar type of system. Reasonable qualitative agreement is obtained when the theory is modified to take account of the finite size of the solids.

Some experimental evidence exists to suggest that the direction of rectification can be reversed when the surfaces are nearly flat. Lewis and Perkins [16] have recently suggested that, with certain special surface geometries, which they claim are typical of flat ground surfaces, thermal strain could also account for this phenomenon. It is shown in section 9 that this is not possible with the usual boundary conditions, but that for this type of geometry there is no steady state solution. A similar result is obtained for the contact of two large solids at a limited contact area, if the applied load is less than a certain limiting value, given by equation (38) for a circular area. It is suggested that this paradoxical result is a consequence of the assumption that any part of the surface must be either in perfect contact or out of contact. In practice, intermediate states will occur in which the local pressure is very light and a finite temperature difference exists locally at the interface. This modified boundary condition introduces the possibility of reversed rectification with certain geometries, but the characteristics of Lewis and Perkins' experimental results are not adequately explained by such a theory. There also exists some evidence of thermal rectification between solids of similar materials which cannot be attributed to thermal distortion.

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APPENDIX I

The integral in equation (5) may be written

$$I_1 = \int_0^s\int_0^{2\pi} \frac{\mathrm{d}r\,\mathrm{d}\theta}{\sqrt{(r^2 - 2rs\cos\theta + s^2)}} + \int_s^b\int_0^{2\pi} \frac{\mathrm{d}r\,\mathrm{d}\theta}{\sqrt{(r^2 - 2rs\cos\theta + s^2)}} \tag{A.1}$$

with a change of variable this becomes

$$I_1 = \int_0^1\int_0^{2\pi} \frac{\mathrm{d}x\,\mathrm{d}\theta}{\sqrt{(1 - 2x\cos\theta + x^2)}} + \int_{s/b}^1\int_0^{2\pi} \frac{\mathrm{d}y\,\mathrm{d}\theta}{y\sqrt{(1 - 2y\cos\theta + y^2)}} \tag{A.2}$$

$$\text{from GR: 3.674.1.} = \int_0^1 4K(x)\,\mathrm{d}x \quad \int_{s/b}^1 \frac{4K(y)\,\mathrm{d}y}{y} \tag{A.3}$$

As $y \to 0$, $K(y) \to \pi/2 + 0(y^2)$.
Thus, if $s \ll b$,

$$I_1 = \int_0^1 4K(x)\,\mathrm{d}x + \int_0^1 \frac{[4K(y) - 2\pi]\,\mathrm{d}y}{y} + \int_{s/b}^1 \frac{2\pi\,\mathrm{d}y}{y}. \tag{A.4}$$

This may be evaluated to give

$$I_1 = 8G + 4\pi\log 2 - 8G - 2\pi\log(s/b)$$

(GR: 6.141.1 and 6.142)

$$= 2\pi\log(4b/s). \tag{A.5}$$

APPENDIX II

The integral in equation (11) may be written

$$I_2 = \int_0^a\int_0^{2\pi} \frac{\{\log(4b/s) - \frac{1}{2}\log(1 - 2x\cos\theta + x^2)\}\,r\,\mathrm{d}r\,\mathrm{d}\theta}{\sqrt{(a^2 - r^2)}} \tag{A.6}$$

where $x = r/s$.
i.e.

$$I_2 = 2\pi a\log(4b/s) - \tfrac{1}{2}\int_0^a\int_0^{2\pi} \frac{\log(1 - 2x\cos\theta + x^2)\,r\,\mathrm{d}r\,\mathrm{d}\theta}{\sqrt{(a^2 - r^2)}}. \tag{A.7}$$

The remaining integral is zero in the range $0 < x < 1$ (i.e. $0 < r < s$) (see GR: 4.224.14). Thus, if $a < s$, we have

$$I_2 = 2\pi a\log(4b/s). \tag{A.8}$$

However, if $a > s$, I_2 contains the additional term

$$I_3 = -\tfrac{1}{2}\int_s^a\int_0^{2\pi} \frac{\log(1 - 2x\cos\theta + x^2)\,r\,\mathrm{d}r\,\mathrm{d}\theta}{\sqrt{(a^2 - r^2)}}$$

$$= -\int_s^a \frac{2\pi\log(r/s)\,r\,\mathrm{d}r}{\sqrt{(a^2 - r^2)}} \tag{A.9}$$

(by GR: 4.224.14).
Integrating by parts and using GR: 2.275.3 we get

$$I_3 = 2\pi\{\sqrt{(a^2 - s^2)} + a\log[a/s - \sqrt{(a^2/s^2 - 1)}]\} \tag{A.10}$$

and for $a > s$,

$$I_2 = 2\pi[a\log\{4b/s[a/s - \sqrt{(a^2/s^2 - 1)}]\} + \sqrt{(a^2 - s^2)}] \tag{A.11}$$

which is the required result.

The average value of heat flow through the contact area is

$$q = \frac{Q}{\pi a^2} = \frac{2K_0(T_1 - T_2)}{\pi a}. \tag{A.12}$$

If this equation is used in place of equation (9) we get

$$\lambda_2 = \frac{C_2(T_1 - T_2)K_0}{\pi^2 a}\int_0^a\int_0^{2\pi}\{\log(4b) - \log[\sqrt{(s^2 - 2rs\cos\theta + y^2)}]\}\,r\,\mathrm{d}r\,\mathrm{d}\theta. \tag{A.13}$$

This expression may be evaluated using GR: 4.224.14 to give

$$\lambda_2 = \frac{C_2(T_1 - T_2)K_0 a}{\pi}\{\log(4b/a) + \tfrac{1}{2}(1 - s^2/a^2)\} \tag{A.14}$$

for $0 < s \leqslant a$ and

$$\lambda_2 = \frac{C_2(T_1 - T_2)K_0 a}{\pi}\log(4b/s) \tag{A.15}$$

for $s > a$.

L'EFFET DE DISTORTION THERMIQUE SUR UNE RÉSISTANCE

Résumé—On traite mathématiquement l'effet de distortion thermique sur la résistance thermique de contact entre deux solides semi-infinis à propriétés différentes. Il est constaté un bon accord avec les observations expérimentales de rectification thermique due à Clausing. On montre que des problèmes de ce type n'ont pas parfois de solution établie et on suppose que ceci est dû à la nature discontinue de la condition limite de contact thermique. On donne des expressions représentant les déplacements de la surface dûs au point d'état permanent et aux sources de chaleur circulaires qui pourraient être d'application plus générale.

DER EINFLUSS THERMISCHER VERZERRUNG AUF DEN ÜBERGANGSWIDERSTAND

Zusammenfassung—Die Wirkung der thermischen Verzerrung auf den thermischen Übergangswiderstand zwischen zwei halbunendlichen Körpern aus verschiedenen Materialien wird mathematisch behandelt. Eine gute Übereinstimmung mit experimentellen Beobachtungen der thermischen Ausrichtung nach Clausing ist erreicht worden. Es wird gezeigt, dass Probleme dieser Art gelegentlich keine stationäre Lösung haben, und es wird angenommen, dass diese Erscheinung der diskontinuierlichen Natur der Randbedingung bei thermischem Kontakt zugeschrieben werden kann. Es werden Formeln angegeben für die Oberflächenverschiebung, infolge stationärer Punktquellen und kreisförmiger Wärmequellen. Sie können für eine allgemeinere Anwendung brauchbar sein.

ВЛИЯНИЕ ТЕРМИЧЕСКОЙ ДЕФОРМАЦИИ НА КОНТАКТНОЕ СОПРОТИВЛЕНИЕ

Аннотация—Предлагается математическое описание влияния термической деформации на тепловое контактное сопротивление между двумя полуограниченными твердыми телами из различных материалов. Получено хорошее соответствие с экспериментальными результатами Клаузинга. Показано, что задачи этого типа иногда не имеют стационарного решения, и сделано предположение, что это происходит за счет прерывистого характера граничных условий для теплового контакта. Предлагаются выражения для перемещения поверхности благодаря стационарным точечным и круговым тепловым источникам, которые могут иметь более общее применение.